METHODS FOR ESTIMATING DAILY SOLAR RADIATION USING CLIMATOLOGICAL DATA

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ABSTRACT

Techniques for estimating daily insolation by determining extra terrestrial sunshine and data on sunshine hours or cloud cover are discussed. Using one year of fairly good records of global radiation and sunshine hours for two stations in The Gambia, at Basse (Lat. 13° 33', Long. 14° 54') and Sapu (Lat. 13° 33', Long. 14° 54') correlation coefficients are derived for the Prescott equation using regression analysis and a computer program. These "constants" are shown to undergo slight variations with location and according to the prevailing season, ie rainy and dry seasons. These are compared with other estimates for West Africa, other parts of Africa, Europe, Asia, USA and Australia.

DETERMINING EXTRA-TERRESTRIAL RADIATION

The total amount of energy radiated by the sun can be determined if the effective absolute temperature of the sun is known. From various research results, this can be assumed to be about 5800 °K (Haltiner and Martin, 1957) and the total energy flux (F) has been calculated as 3.9169×10^{26} watts or 56.146×10^{26} cal min $^{-1}$. Since this amount of energy is also at normal incidence on a sphere concentric with the sun and at a radius equal to the mean sun-earth's atmosphere, it can also be determined by:

$$F = 4\pi R_o^2 I_o \tag{1}$$

Where R_o is the mean sun-earth distance of 1.5×10^{13} cm and Io is the so called solar constant defined in watts cm^{-2} or cal min $^{-2}$. Thus from equation (1) the solar constant can be determined, ie

The solar constant was first introduced by A Pouillet in 1837. Its estimated value varies between 1.946 and 2.0 cal min $^{-1}$ cm $^{-2}$ according to the work of various researchers. In this paper a figure of 1.98 cal min $^{-1}$ cm $^{-2}$ will be assumed in the subsequent calculations.

By use of I_o, the total amount of energy received by the part of the earth/atmosphere system facing the sun can be determined according to the following formula

$$E_s = \pi r_e^2 I_o \tag{2}$$

Where r_e is the radius of the earth equal to 6.37×10^8 cm. This gives the total energy received as 2.55×10^{18} cal min $^{-1}$. At the top of the earth's atmosphere the amount of radiation, more specifically, depends on the orientation of a receiving surface in relation to the sun's normal incident radiation. This in turn depends on the time of the year, time of the day and the latitude over which the surface is located. From Figure 1, it is clear that the amount of radiated energy through a surface A_n is equal to the energy through A_h since no attenuation of this energy took place before it enters the earth's atmosphere, thus;

$$I_o \; A_n = R_s^1 \; A_h \tag{3}$$

Where R_s^1 is regarded as the instantaneous extra-terrestrial radiation. It is to be noted also from this figure that

$$\frac{An}{Ah} = \cos Z \tag{4}$$

Where Z is the solar zenith angle, ie the angle subtended by the local vertical originating from the centre of the earth and the line parallel to the normal rays of the sun. Since the earth follows an elipse in its movement around the sun, the value of I_o is modified according to the square of the ratio between the mean sun-earth distance (R_o) and the instantaneous sun-earth distance (R). Equation (3) can therefore be revised accordingly, ie

$$R_s^1 = I_o \left(\frac{R_o}{R}\right)^2 \cos Z \tag{5}$$

From figure R and by use of spherical geometry, it can be proven that

$$\cos Z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \tag{6}$$

Where ϕ is the latitude of the point of observation, is the solar declination and h the hour angle or the angle through which the earth rotates to bring the point p under the direct rays of the sun. The hour angle is defined to be zero at solar noon and exactly 90° at sunrise or sunset during equinoxes. Substituting equation (6) in equation (5) and integrating over a period of 24 hours ie complete rotation of the earth, R_s can be defined as follows;

$$R_s = I_o \left(\frac{R_o}{R}\right)^2 \int_{-h}^{h} (\sin \phi \sin \delta + \cos \phi \cos \delta \cosh h) \, dt \tag{7}$$

Since $\frac{dh}{dt}$ = W, which is the angular velocity of the earth given in radians such that 2 radians is covered in a day, equation (7) can now be rewritten as;

$$R_s = I_o \left(\frac{R_o}{R}\right)^2 \int_{-h}^{h} (\sin \phi \sin \delta + \cos \phi \cos \delta \cos h) \frac{dh}{W} \tag{8}$$

Which yields after integration

$$R_s = \frac{1440}{\Pi} I_o \left(\frac{R_o}{R}\right)^2 (h \sin \phi \sin \delta + \cos \phi \cos \delta \sin h) \text{ cal cm}^{-2} \text{ day}^{-1}$$

In which h is expressed in radians (0.5 radians = 90°) in the first term of the equation. From equation (6) and by definition of h, it can be shown that at sunrise or sunset for all latitudes Cos = 0, which means that

$$\cos h = \frac{-\sin\phi \ \sin\delta}{\cos\phi \ \cos\delta} = -\tan\phi \ \tan\delta \tag{10}$$

In solving equation (9), the solar declination can be estimated by various methods. A series solution (Paltridge and Platt, 1976) has been derived as follows;

$$\delta = 0.006918 - 0.399912 \cos\theta_o + 0.0702575 \sin\theta_o - 0.006758 \cos 2\theta_o + \tag{11}$$

$$0.000907 \sin 2\theta_o - 0.002697 \cos 3\theta_o + 0.001480 \sin 3\theta_o$$

Where = 2 d/365(6) and d being the number of the day in the year from 1 to 365(6). Equation (11) estimates with a maximum error of 0.0006 radians ( ).
can also be estimated using the following relationships developed by the US Department of Energy, Division of Solar Energy, ie

$$\delta = 23.45 \sin(1.008(n-80)) \quad 1 \le n \le 80$$

$$= 23.45 \sin(0.965(n-80)) \quad 81 \le n \le 266 \tag{12}$$

$$= -23.45 \sin(0.975(n-266)) \quad 267 \le n \le 365(6)$$

where n is the number of the day in year from 1 to 365(6). The day given against a particular declination may vary by one or two days given this approach. The solar constant modifier ( ) can also be calculated using a series expansion (Paltridge and Platt, 1976);

$$\left(\frac{R_o}{R}\right)^2 = 1.000110 + 0.034221 \cos\theta_o + 0.001280 \sin\theta_o + \tag{13}$$

$$0.000719 \cos 2\theta_o + 0.000077 \sin 2\theta_o$$

ESTIMATING GLOBAL RADIATION AT THE EARTH'S SURFACE

The solar constant term I_o is only useful for estimating extra-terrestrial radiation. At the surface of the earth, the energy received is largely attenuated. The attenuation of solar radiation by the earth's atmosphere is complex and depends on parameters that are not only difficult to estimate but also subject to considerable spatial and temporal variations. Furthermore, the situation is complicated by the presence of clouds which readily reflect back into space a portion of the incoming radiation.

Because direct measurement data are not always available, certain semi-empirical approaches have been suggested. One which is of interest is the Prescott Equation (1940). This employs calculated extra-terrestrial radiation values and a ratio of measured sunshine hours (n) and the daily possible sunshine hours (N) obtainable from equation (10). Prescott's equation is in the form

$$R_E = R_S(a + b \ {}^{n}/_{N}) \tag{14}$$

where R_E is the estimated global radiation, a and b are empirical constants which are obtained from regression analysis between the ratios R_g/R_S and n/N where R_g is measured global radiation.

When daily sunshine hours data are not available, mean fractional cloud cover data can be used. Black (1956) using cloud cover values for 88 European stations established a quadratic relationship between cloud cover (C in oktas) and the ratio of measured global radiation and calculated extra-terrestrial radiation, in the following form, ie

$$R_g/R_s = 0.803 - 0.340\,(C/8) - 0.458\,(C/8)^2 \qquad (15)$$

Davies (1965, 1967) derived the constants of equation (14) for some stations in West Africa (Table 1). He found that both the values of a and b vary with location and time of the year; a decreases and b increases toward higher latitudes. He observed that in general a depends only on the displacement of the regression line which in turn depends on the scatter of the plotted points. There were more scattered points during the wet season than during the dry season. Coefficient b defines the rate of change of R_g/R_s with changes in cloudiness and this declined from a December maximum to a September minimum. Other researchers have derived Prescott's constants for other locations around the world (Table 2).

DERIVING THE CONSTANTS OF THE PRESCOTT'S EQUATION FOR STATIONS IN THE GAMBIA

As noted earlier, the use of equation (14) for estimating incoming global radiation requires prior determination of a and b for a given location. For the Gambia, data of global radiation and sunshine hours from two stations, viz Sapu and Basse, have been used in a computer program for the derivation of these constants. The data covered a period of one year.

By determining extra-terrestrial radiation from equation (9) and total possible sunshine hours from equation (10), regression analyses on the ratios R_g/R_s and n/N were performed and thus produced the values as indicated in Table 3. In general, a relatively high correlation coefficient resulted between daily values. The values of a are higher during the wet season (June to October) while the values of b attain their minimum during this same season. In general, better correlation coefficients resulted during the dry season (November to May).

After determining the Prescott constants based on correlation for a monthly period, global radiation for each day of that month was then estimated by using equation (14) (Table 4). The cloud cover and equation (15) were also used to estimate global radiation for comparative reasons.

CONCLUSIONS

In Table 4 is reproduced a sample of the computer results for the station of Sapu for the month of January. The estimated results of global radiation R_E and R_C, respectively using sunshine hours and cloud cover, both provided good estimates of the actual measured values. The values estimated with cloud cover on average were higher by about 5%. In general, it is to be recommended that the Prescott constants be determined for each month so as to accommodate likely climatic variations during the course of the year.

REFERENCES

Black JN (1956) The distribution of solar radiation over the earth's surface. Arch Met Geoph Biokl, Ser B, 165-189.

Brutsaert W (1982) Evaporation into the atmosphere; theory; history and applications. Reidel, Boston.

Coulson K L (1975) Solar and terrestrial radiation. Academic Press, New York.

Davies JA (1965) Estimation of insolation for West Africa. Quart J R Met Soc, 91, 359-363.

Davies JA (1967) A note on the relationship between net radiation and solar radiation. Quart J R Met Soc, 93, 109-115.

Haltiner GJ, Martin FL (1957) Dynamical and physical meteorology. Mcgraw-Hill, New York.

Jobe L (1982) An investigation of actual regional evapotranspiration from the Sudano-Sahelian Zone of West Africa. Theses (M.Sc.), Cornell University, New York.

Paltridge W, Platt CMR (1976) Radiative Processes in meteorology and climatology. Elsevier, New York.

Robinson N (1966) Solar radiation. Elsevier, New York.

Sellers D (1965) Physical climatology. University of Chicago Press, Chicago.

Snedecor GW, Cochran WG (1967) Statistical methods. Iowa State University Press, Iowa.

TABLE 1: R_g/R_s versus $^n/_N$ for different data groupings (Davies, 1965)

GROUPINGS	a	b	r (Correlation Coefficient)
All data	0.19	1.60	0.86
By station			
Benin	0.28	0.33	0.84
Kano	0.26	0.54	0.83
Accra	0.30	0.37	0.82
Fort Lamy	0.43	0.32	0.72
Niamey	0.11	0.72	0.82
By month			
January	-0.04	0.88	0.99
February	-0.04	0.88	0.92
March	0.07	0.80	0.96
April	0.08	0.82	0.97
May	0.12	0.73	0.93
June	0.19	0.61	0.87
July	0.20	0.64	0.93
August	0.20	0.60	0.86
September	0.26	0.50	0.92
October	0.17	0.62	0.92
November	0.07	0.74	0.94
December	-0.12	0.99	0.88

TABLE 2: Some values of the constants in Prescott's Equation for different locations (Brutsaert, 1982)

Location	Latitude	Period	a	b
Accra (Ghana)	6°	Monthly	0.30	0.37
Kano (Nigeria)	12°	Monthly	0.26	0.54
Kunamura (W. Australia)	16°	Daily	0.334	0.431
Delhi (India)	29°	Weekly	0.31	0.46
Tateno (Japan)	36°	Monthly	0.25	0.54
Dodge City (Kansas, USA)	38°	Daily	0.230	0.542
Cleveland (Ohio, USA)	41°	Daily	0.188	0.539
Madison (Wisc. USA)	43°	Daily	0.208	0.530
De Bilt (Netherlands)	52°	Daily	0.22 $\pm$ 0.011	0.50 $\pm$ 0.02
Rothamsted (England)	52°	Monthly	0.18	0.55
Matanuska-Anchorage (Alaska, USA)	61°	Daily	0.261	0.465

TABLE 3: Computed constants of the Prescott's equation and correlation coefficients based on 1980 data

Station	Month	a	b	r (correlation coefficient)
Sapu	January	0.250	0.399	0.91
(Lat. 13.55°	February	0.264	0.387	0.90
Long. 14.96°)	March	0.277	0.432	0.86
	April	0.237	0.392	0.90
	May	0.288	0.393	0.87
	June	0.276	0.380	0.90
	July	0.302	0.360	0.89
	August	0.303	0.362	0.88
	September	0.288	0.371	0.89
	October	0.316	0.379	0.83
	November	0.280	0.384	0.89
	December	0.238	0.405	0.91
Basse	January	0.255	0.380	0.87
(Lat. 13.332°	February	0.315	0.326	0.84
Long. 14.22°)	March	0.269	0.398	0.86
	April	0.328	0.359	0.75
	May	0.325	0.368	0.75
	June	0.384	0.271	0.71
	July	0.293	0.347	0.88
	August	0.337	0.303	0.85
	September	0.477	0.148	0.53
	October	0.332	0.369	0.73
	November	0.270	0.369	0.87
	December	0.125	0.476	0.84

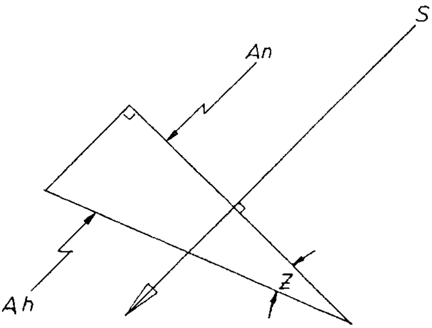
TABLE 4: Sample of the computation results, Sapu, Lat. 13.55°, January 1980

Day	δ radians	R_S cals/cm² day	R_g cals/cm² day	N hrs	n hrs	n/N	R_g/R_S	R_C cals/cm² day	R_E cals/cm² day	
1	-.401069	710.192	418	11.2179	9	.802291	.588573	547	404	
2	-.399557	711.183	404	11.2212	8.9	.793143	.568068	536	402	
3	-.397913	712.251	355	11.2248	7.3	.650348	.498420	348	362	
4	-.396138	713.394	80	11.2286	0	0	.112140	80	178	
5	-.394233	714.613	364	11.2328	.9	.080123	.509367	111	201	
6	-.392199	715.904	225	11.2372	2.5	.222476	.314288	111	242	
7	-.390037	717.268	243	11.2419	2.6	.231279	.338785	141	245	
8	-.387747	718.704	158	11.2468	.4	.035566	.219840	111	190	
9	-.385331	720.209	293	11.252	3.5	.311056	.406826	142	269	
10	-.382790	721.783	487	11.2575	10.3	.914947	.674718	544	443	
11	-.380125	723.424	446	11.2632	10.2	.905604	.616513	483	442	
12	-.377337	725.130	413	11.2692	9.4	.834134	.569553	424	422	
13	-.374427	726.901	424	11.2754	9.7	.860280	.583298	574	431	a=0.250
14	-.371397	728.734	402	11.2819	9.8	.868651	.551641	502	434	b=0.399
15	-.368249	730.628	371	11.2886	8.5	.752975	.507782	325	401	r=0.91
16	-.364983	732.581	352	11.2955	7.1	.628570	.480493	276	366	
17	-.361600	734.592	418	11.3026	9.7	.858207	.569023	506	435	
18	-.358104	736.658	409	11.3100	9.7	.857647	.555210	592	436	
19	-.354494	738.778	436	11.3176	9.8	.865906	.590164	569	439	
20	-.350773	740.950	430	11.3254	10.2	.900628	.580336	453	451	
21	-.346942	743.172	430	11.3334	10.2	.899991	.578601	363	452	
22	-.343002	745.442	465	11.3417	10.2	.899339	.623791	408	453	
23	-.338957	747.757	493	11.3501	10.9	.960345	.659305	516	472	
24	-.334807	750.117	480	11.3587	10.5	.924402	.639900	533	464	
25	-.330553	752.518	474	11.3675	10.4	.914890	.629885	479	462	
26	-.326199	754.960	388	11.3765	8.1	.711996	.513935	391	403	
27	-.321745	757.439	421	11.3856	9.9	.869518	.555821	337	452	
28	-.317194	759.953	493	11.3949	10.4	.912685	.648724	584	466	
29	-.312547	762.501	486	11.4044	10.5	.920694	.637376	574	470	
30	-.307806	765.081	486	11.4141	10.6	.928676	.635227	614	474	
31	-.302973	767.689	487	11.4239	10.2	.892864	.634371	590	465	

(δ : solar declination; R_S: extra-terrestrial radiation; R_g: measured global radiation; N: possible hours of sunshine;
n: measured sunshine hours; R_C: radiation estimated with cloud cover

FIGURE 1: Radiation at the top of the atmosphere

(A_n: surface normal to the sun's rays; A_h: surface parallel with the earth's surface; S: direct radiation from the sun; E: earth's surface; A: top of the atmosphere; Z: solar zenith angle).



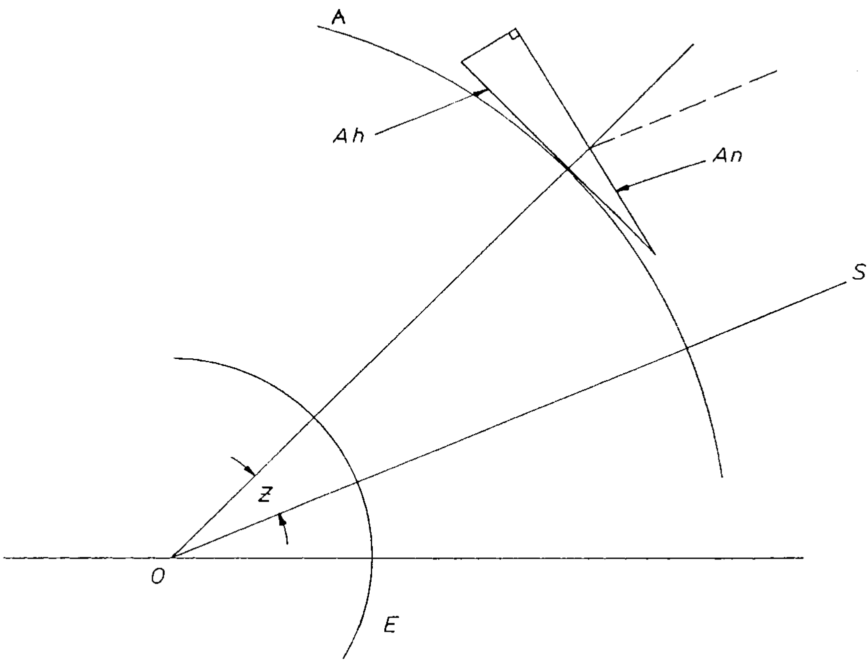


FIGURE 2: Relationship of the solar zenith angle (Z)

Latitude (ϕ), Declination (δ) and Hour Angle (h). (Sellers, 1966)