CHAPTER VIII

Resources for Learning Mathematics

(Including textbooks, films, radio, television, programmed learning, etc.)

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Introduction

1. I am both pleased and mystified by the invitation of the Organising Committee to address this Conference on the topic "Resources for Learning Mathematics". I am pleased, because the invitation has given me the stimulus to gather together and think about some of the many books and papers on this topic, and because of the unexpected opportunity to meet and exchange ideas with so many Commonwealth leaders in mathematical education; I am mystified, because I do not believe that I have ever made any significant contribution to the development, use, or assessment of any of the resources mentioned in the title of my topic, and I cannot imagine why I should have been asked to speak on this topic to a conference of experts.

2. It is inevitable that some of what I say will overlap with other prepared papers. This is so because the training of teachers, and evaluation, are the topics of other plenary sessions. But teachers are among the most important of our resources, and the usefulness of a teacher as a resource is obviously related to his training; moreover the effectiveness of any resource must be determined by evaluation, and it is difficult to divorce the evaluation of programs from the evaluation of resources.

3. It is also inevitable that, due to the need to prepare this paper in a few brief months, there will be significant omissions of important material. This, however, is no cause for concern, and can be considered as a stimulus to the appropriate Working Party to put things right by filling in the gaps.

Terms of Reference

4. When approached to prepare this paper on resources, my "brief" consisted solely of the assigned title, and a suggestion that I give relatively less attention to the more sophisticated and expensive new technological developments. I have interpreted my brief very liberally, taking advantage of the "etc." which came with the title, and also of the fact that the title uses the expression "*learning* mathematics", rather than the expression "*teaching* mathematics". Thus in addition to the specific resources listed in the title, I intend to include our environmental resources (physical, biological, socio-economic, technological, cultural, etc.); books other than textbooks; computers; and that most important resource, the teacher.

This extended list is by no means complete (we haven't even listed "money"!); nor, of course, are the items in it independent.

Educational Media

5. Many of the resources I have mentioned above (possibly all of them) are included in the currently popular term "media". Perhaps I should have said "educational media", but the adjective is probably redundant: surely all media are capable of being used for educational purposes. In this connection I would like to refer you to the recent book *Understanding Media*[1] by Marshall McLuhan. The title notwithstanding, I do not know anyone who claims to understand a great deal of what McLuhan is trying to say, and yet I have the feeling that it is important, and that it will stimulate further thought and work which will be highly relevant to the topic of this paper.

Resources and Learning Theory

6. In comparatively recent times psychologists have made great strides in their studies of human personality; anthropologists and psychologists have made us very aware of the relationship between personal development and environment; scientists and technologists have steadily increased our ability to modify our environment; and educational researchers have been actively exploring the relationship of environment and personality to the learning process. A variety of learning theories have been formulated, and serious attempts have been made to evaluate them. Some of these theories (e.g. the theories of B.F. Skinner: see below), have had a considerable influence on the development and use of some of the most "modern" learning resources: in particular, programmed instruction. No doubt this situation will continue, with our understanding of the learning process, and a more sophisticated and successful use of resources, developing hand in hand.

Resources for learning Mathematics

7. We have not come to this Conference in order to speculate about the uncertain future, but rather to face up to the realities of the present. We wish to exchange information, experiences and opinions on those resources for learning mathematics which already exist, or are likely to exist in the near future. In introducing these discussions, it therefore falls to me to say something about a number of specific resources. In discussing these separately, I will attempt to say a little about their current state of development and use, and to include opinions as well as facts, in the expectation of provoking some of you to disagree with me. The list is not intended to be categorical, nor are the items in it independent of one another.

Our Natural Environment

8. All of the specific resources which we shall consider below form part of our total environment. This environment is by no means the same for all of us – in fact every one of us is a genetically unique individual, and each of us is subject from birth to a personal and unique set of environmental factors. Thus at every stage of our development our potentiality for learning (in mathematics as in all else) is unique, and, ideally, this should be exploited by a unique use of available resources. However all existing systems of education operate by considering categories of individuals, grouping them by the use of such criteria as geographical location, age,

sex, apparent success in previous education, and so on. It is convenient to consider a rough division of our total environment into that part which is natural, and that part which is man-made. The "mathematical content" of our natural environment is much the same for all of us, wherever we happen to be located on this little planet. The shape of the sun; the apparently changing shape of the moon; daily and seasonal patterns of shadows; regularity and symmetry in crystals, leaves, flowers, and seeds; experiences with water, such as reflections, waves and ripples; – these are freely available to all. We know that many of these things have had a significant effect on the historical development of mathematics, and it is not unreasonable to believe that we should be able to exploit them as resources for the learning of mathematics.

9. Although it is probably not relevant to the topic of this paper, we should remind ourselves that there is a deeper relationship between mathematics and the natural environment, and that until very recently this environment has been virtually unchanged during the evolution of man. There is little doubt that among those genetic variations of prehistoric man (and his ancestors) which had survival value in relation to the environment, there must have been variations related to abilities which we would consider to be essentially mathematical. In other words, the fact that man has evolved as an animal with considerable mathematical capability probably reflects the value which this type of ability has had for survival in a competitive environment.

Our Man-Made Environment

10. Under this heading we include all aspects of our total environment which are, directly or indirectly, the creation of man. This includes not only those specific products of our manufacturing technology which are mentioned in the title of this paper, but also such other man-made creations as our homes; our cities; our social, political and economic systems (including our use of money); our many devices for the transportation of goods, information and ideas; and, of course, the many useful and useless gadgets which flow from the application of our advancing science and technology. The natural environment has played a well-known and significant role in the historical development of mathematics, but the man-made aspects of our environment (and our need to understand and further develop them) have now assumed a far greater significance in relation to current mathematical developments, and to current curriculum developments. While we seem to be moving slowly in the direction of greater uniformity in the technological aspects of our societies, there are still very great differences from one place to another. Whether or not this implies the need for differences in mathematics curriculum will, no doubt, be discussed in other sessions; but there seems little doubt that these environmental differences deserve serious consideration in relation to the use of resources for implementing whatever curriculum is used.

11. Let me give a simple example. In my country, it has been pointed out to me that the mathematical progress of children in rural areas is indistinguishable from that of city children through the primary grades, where the mathematics studied is almost wholly arithmetic; but that apparently significant differences begin to appear at the secondary level, with urban children achieving greater success than rural children. It is conjectured that the environment of urban children (includ-

ing, of course, the attitude of parents, friends and especially, the peer group) makes the acquisition of mathematical knowledge more desirable, and apparently more obviously relevant to the needs of the community. The same curriculum is almost certainly needed for all of these children, in view of the uncertainty concerning their future careers. But one naturally wonders whether or not the different environmental influences could be compensated for by the use of different teaching strategies.

12. The profound influence of our cultural environment on our personal development is, of course, well known to psychologists and to anthropologists but we sometimes tend to overlook it in relation to the learning of such a universal and apparently culture-free subject as mathematics. It is my personal belief that not enough attention has been paid to cultural differences, both in the design of curricula, and in the use of various resources for learning mathematics. It might be hopelessly visionary to contemplate a time when the design of mathematics curricula and the use of learning resources will be fully adapted to the needs and potentialities of each individual, but it is not quite so unrealistic to suggest that more attention (especially in the use of resources) should be paid to some of the more obvious cultural differences. This implies, for example, that books designed for use in one society should be "culturally translated", as well as verbally translated, in order to adapt them for the needs of another – an innovation which has already been found worthwhile for culturally distinguishable groups within one country, as well as between countries with obvious cultural differences. (See for example[2], and many other American reports on programs for culturally disadvantaged children). Of course, many other dichotomies are possible in addition to such well known ones as rural-urban, affluent-poor, and culturally "normal"-culturally deprived. One which has received a great deal of attention in recent years is related to the relative state of technological advancement, classifying countries rather roughly as "developed" and "developing". I recently read a charming book[3] about the experiences of two teachers from one of the most technologically developed countries (the United States of America) in teaching elementary mathematics to children from a particular tribal group in Liberia. This delightful and thought-provoking study has implications which go far beyond the immediate experiences of the authors: it suggests most strongly that the teacher should understand the culture in which he is working, and how this affects not only the suitability of what is taught (the curriculum) but also the choice of a teaching strategy (including the use of learning resources) which is likely to prove most effective.

13. This attention to cultural differences is important in time, as well as in place: the cultural environment of any part of the world varies in time, and in most areas the position today is very different to what it was fifty (or even twenty) years ago. Dr. Robert Davis has drawn attention to the tremendous significance of these time-linked cultural changes, in his fascinating and provocative booklet[4], *Mathematics: The Changing Curriculum.* This booklet is essential reading for anyone who is interested not only in seeing where we are (in mathematical education), but who is also interested in seeing some of the directions in which we are heading. Dr. Davis points out that the technology of the 1960's and the use of mathematics in it, are vastly different to what they were in the 1940's, and he suggests that all of

our recent "reform" efforts, even if successful, might only result in making our curricula relevant to the needs of the 1940's. He suggests that the technological development of (American) society requires a rate of change of mathematics curriculum and learning procedures which is considerably greater than the present rate.

14. I shall not carry further this discussion of environmental factors, but I hope that what I have said will make clear my belief that our man-made environment has had, and will continue to have, a great influence on the developing nature of our mathematics, on the choice of curriculum, and on the choice of strategy for teaching (learning) this curriculum, and my strong conviction that both the choice of curriculum and the use of available learning resources should take into account the nature, and the changing nature, of the cultural background of our students, and of the future environment in which they will live. If we can discover how to do this successfully, then it will be possible to exploit the environment itself much more than we do as a resource for learning mathematics.

15. Of course, it is rather easy to assert what should be done. We might all agree on the importance of cultural factors in curriculum design and in learning strategy, but find great difficulty in turning this agreement into action. However, the report[2] referred to earlier, indicates that some progress is being made in one country (the United States) in the understanding and amelioration of the cultural deprivation which results from relative poverty. I recently read in the Annual Report (1967) of the Carnegie Corporation of New York, of a quite different project (a joint project of Syracuse University and Makerere College) designed to discover whether or not current psychological theories of child development and learning (which have been predominantly based on studies of children in Western societies) are, in fact, applicable to non-Western cultures. Studies such as these indicate that some progress is possible, and they hold much promise for the future.

Specific Resources: The Teacher

16. During the past fifteen or twenty years there has been an almost unprecedented amount of activity aimed at the "improvement" of school mathematics. Initially much of this activity was directed towards curriculum reform, with such landmarks as the creation of the Commission on Mathematics in 1955, and the publication of its report[5] in 1959; the establishment of the School Mathematics Study Group, in 1958; the Royaumont Seminar (1959) of the Organisation for European Economic Co-operation, and its controversial report[6] *New Thinking in School Mathematics;* the call to arms by Professor Bryan Thwaites (1961) in his Inaugural Address[7], leading to the Southampton Mathematics Conference[8], and to the School Mathematics Project; the imaginative work of the Cambridge Conference on School Mathematics (1963) and its stimulating report[9] *Goals for School Mathematics;* and, more recently (1966) the Ditchley Conference[10], a joint Conference of the School Mathematics Project and the Cambridge Conference.

17. This listing is, of course, very incomplete. Reform movements exist in virtually every country, and there can scarcely be a school anywhere whose mathematics program has not been affected by the activity of the last decade. (A fuller

picture is given by the recent UNESCO publication[11] *New Trends in Mathematics Teaching).* In most of the above mentioned reports there is relatively little mention of the role of teachers and other resources. However it was early realised by many of the leaders of the reform movements that the best-conceived curriculum would founder if due attention were not paid to the crucial role of the teacher and to the improvement of teachers themselves, as well as the improvement of other resources such as textbooks and teachers' guides.

18. The events of the last ten years have fully justified the initial concern (which is evident in many of the publications referred to above) that a major barrier to the successful implementation of any new curriculum would be the mathematical and pedagogical competence of the teacher. I am sure that many of us who are here at this Conference will have independently reached the conclusion that it is the teacher on whom we must concentrate much of our attention, if genuine progress is to be made. This is not really surprising. For it is the teacher (who is often referred to as "a filter, through which mathematical ideas percolate to the student"; or as "a mirror – sometimes cloudy! – in which the student perceives mathematical concepts") who is still the main channel for the communication of mathematical ideas to the student, no matter what curriculum is being studied. This statement is as true for classes using the many fine new textbooks that have recently appeared, as it is for classes using older books or having no textbooks at all. And, as far as we can tell at present, the role of the teacher is likely to be just as significant (even if different in detail) in conjunction with such "automated" teaching methods as programmed learning and computer assisted learning.

19. It is my own conviction that the teacher is, in many ways, the critical classroom resource, and that the teacher sets a limit to the effective use of every other resource – textbooks, films and filmstrips, programmed materials, attribute blocks, calculators, overhead projectors, number rods and other analogy devices, mathematical games and puzzles, and the many other "teaching aids" which are appearing in our classrooms. For this reason I look forward with interest to the workshop discussions on the vital topic of teacher selection and training. Unfortunately it is extremely easy to convince ourselves of the crucial role of the teacher, but it is vastly more difficult to know just what to do about it. We know that the successful teacher must have an adequate knowledge and understanding of the relevant mathematics, and as much skill and understanding of the processes of learning and of pedagogy as we can currently provide: these are the traditional components of teacher training. But there is much more to it than this. Most of us will recall personal learning experiences in mathematics (and in other subjects) where we became excited about a particular topic, and this excitement acted as a catalyst in the learning of the topic, to the extent that we were not conscious at all of any "effort" to learn. We would like to be able to excite all of our teachers about their subject and its teaching, and to have them communicate this excitement to their students. Perhaps one day we shall discover how to do this.

Books

20. Let me confess at the outset what will probably become apparent as I proceed: that I am a book lover. Books have been with us for a very long time,

and their role as carriers of various aspects of our human culture is well known. (Consider, for example, the historical significance of such books as the Bible, the Koran, and Euclid's Elements). The two great storehouses for the accumulation and transmission of mathematical knowledge have been people and books.

21. In considering the significance of books as a resource for learning mathematics, it is necessary to remind ourselves of the dramatic change in the status of books brought about by the development of modern, movable-type printing in Western Europe in the fifteenth century. This technological breakthrough made it possible to produce many identical copies of the same book at a great reduction in cost as compared with earlier methods. It also made possible a rapid development of mass education. The reasons why such a development did not occur for several hundred years are complex, and beyond the scope of this discussion; but it is safe to say that, had printing not been invented, the development of mass education in the last hundred years would have been very different from what it has been. To realise this we have only to reflect on the extent to which our personal development – in mathematics as in other areas – has been assisted by our use of books: textbooks, reference books, journals, and so on.

22. Books and printing have been with us for so long that it might be thought that, except for changes in content, nothing of current significance could be said about the role of books. But I do not believe that this is the case: authors and book publishers have not stood still. One interesting development of the last decade (closely associated with curriculum reform) has been the growing tendency for the use of fairly large groups of writers, rather than the more traditional authorship by one, or by a small number of writers. This group writing is seen in the work of such bodies as the School Mathematics Study Group, the School Mathematics Project, the African (Entebbe) Mathematics Project, and, of course, many others. There is no doubt that such group writing can result in a very critical selection and appraisal of material, although it sometimes runs into problems of style. I am sure that the potentialities of group writing have not been fully realised in most of the recent projects, due to rather severe restrictions of time and money. But, given an adequate supply of both, it seems safe to conjecture that many of the best textbooks of the future will be produced by the co-operative efforts of mathematicians, teachers, learning theorists, and publishers; assisted, of course, by feedback from the students on whom the texts are evaluated. In this connection see the remarks of Robert Davis on "Experimenting with Textbook Style"[4].

23. There have also been significant changes in recent years in the art and technique of book production. For example, consider the attractive use of art work and colour in the books[12] of the Nuffield Mathematics Project: these books are a far cry from the generally drab textbooks of a generation ago. Another innovation has been in the use of transparent overlays; for an example of this see[13]. It is not clear just how much effect a more attractive production can have on the learning of mathematics, quite apart from the aesthetic pleasure which it may give us, but I am inclined to think that it could be significant for some students; it would be interesting to see some well planned educational research on this question.

24. While talking textbooks, there is one matter which simply must be discussed: that is, how significant is any textbook in the learning process for the individual student, other than as a course-guide for the teacher? In my country, until recently, mathematics textbooks for the primary school (and, to a lesser extent, the secondary school) were little else than collections of problems. The "textbook" (if any) was not generally designed to present the subject in an orderly way which could (in theory at least) be read by the student as a supplement to his class instruction. There are signs that this situation is changing significantly, and I will be most interested to see whether these students (who are now being encouraged to read mathematics books at both primary and secondary levels) will impress us, who later teach them at the tertiary level, as being significantly more successful than our present students in their ability to learn mathematics, by themselves, from books and journals.

25. At this stage I hope that you will forgive me if I inject a somewhat more personal note, and tell you of a project on which I have been engaged for the past seven or eight years. About ten years ago, as a result of my increasing involvement with mathematics education in schools, I became aware that most schools in the region where I live (Perth, Australia) had virtually no worthwhile mathematics holdings in their libraries. (A typical situation was for a school to have a few musty and out-dated textbooks, frequently bequeathed by some long forgotten teacher). I also became aware that very few teachers of mathematics had any significant number of mathematics books in their personal libraries. In view of the rapidly growing number of suitable books (for both teachers and schools) which I knew to exist, it seemed to me that I should be able to do something to improve this situation. My first approach was to gather together a list of a little over a hundred suitable books, and I circulated this list to teachers (as members of our Mathematical Association) and to school librarians. After a year or so I made some attempt to assess what effect this had had. Unfortunately it appeared that any improvement was very small indeed. After further discussions with teachers, I suddenly realised that neither teachers nor librarians were likely to buy any significant number of mathematics books which they had no opportunity to inspect in advance. And Perth bookshops (including our University Bookshop) rarely stock very many mathematics books which are suitable for the libraries of schools and of teachers.

26. Having thus clarified my problem, the solution was relatively simple. I set about collecting a "standing display" of such books, housed in a location where they could be readily inspected (including limited borrowing) by any interested teacher. In addition, and with the co-operation of the State Department of Education, these books are regularly circulated (for a limited inspection period) in boxes of 20-30 to any interested school. (You might be interested to know that the State of Western Australia occupies almost a million square miles, and that some schools are over 1,000 miles from Perth. Thus not every teacher can drop in at the University – where the books are normally housed – after school!) Finally, a list of recent acquisitions is sent regularly to all members of the Association.

27. This collection has now grown to about 700 books. In case you are interested in starting such a project for yourselves (and I strongly recommend that such "school mathematics collections" should be established in every major centre) I have arranged

to supply each of you with a fairly complete list of our current holdings. No doubt some of this material will be familiar to you, but I am sure that you will be as surprised as I was just how much is available, and how rapidly this is increasing.

28. Perhaps I should say something about the selection of material for this collection. I began originally with the idea of sending out a brief "review" of each book, with each list of acquisitions. But almost from the beginning I had requests from teachers to put the books in some sort of order of preference; and this worried me, because I was not convinced of the infallibility of my judgment (or of anyone else's), and I did not like the thought that every school might end up with the same little subset of my collection. My concern about possibly playing the role of "censor" soon became irrelevant, because the books came in much faster than I could possibly read and comment on them, and the only books which I have excluded have been quite routine standard texts.

29. As a result of all this, teachers and schools have made their own selections, and these have shown a gratifying diversity. Some of them have certainly bought books which I would never have recommended. But this doesn't worry me, especially as teachers have told me of particular books which have aroused the interest of some of their students, and I have realised that I would have deleted these books in any censoring process. I am more than satisfied if a book which I consider dull, inelegant, and even inaccurate, can arouse the interest of any student in mathematics: interest acts as a powerful catalyst in the learning process.

30. So that some of you can see some of these books for yourselves, I have arranged to have as many of them as possible brought together in Trinidad.* I hope that some of you might be influenced to work towards the establishment of such collections at suitable centres in your own countries, and, if so, my experience might provide you with a starting point. Of course if such a collection is started, it should be kept up-to-date by the regular addition of new material. Among the sources which I find useful in helping to locate suitable books are the reviews, advertisements, and bibliographies in such journals as *The Mathematics Teacher, The Arithmetic Teacher, The Scientific American, School Science and Mathematics, Mathematics Magazine, The Mathematical Gazette, Mathematics Teaching,* and *Teaching Arithmetic.* In addition annotated lists have been published by the Association of Teachers of Mathematics (U.K.), and by the National Council of Teachers of Mathematics (U.S.A.).

Programmed Learning

31. It is fairly safe to say that programmed instruction has been one of the most controversial educational innovations of the past decade. The recent wave of interest and activity in this technique has its origin in the work of S.L. Pressey, who pointed out in two articles published in 1926 *(A Simple Apparatus Which Gives Tests and Scores – And Teaches* and *A Machine For Automatic Teaching of Drill Material;* reprinted in[14], pp. 35-41 and pp. 42-46) that a multiple-choice testing machine also performed a teaching function. However, the roots of the idea are as old as education itself, and are certainly present in the didactic methods of Socrates. In fact,

*These titles are included in the bibliography.

as Kenneth May has pointed out in his masterly studies *Programmed Learning and Mathematical Education*[15], and *Programming and Automation*[16], programmed instruction in the modern specific sense can be considered as a particular instance of educational programming, which he defines as the scheduling and control of student behaviour in the learning process. Thus all educational procedures involve programming (including self-programming) to a greater or lesser extent.

32. Pressey's work does not appear to have made any significant impact at the time of its publication, but this cannot be said of the later work of B.F. Skinner. Following the appearance of Skinner's papers *The Science of Learning and The Art of Teaching* and *Teaching Machines* (which appeared in 1954 and 1958 respectively; these are reprinted in[14], pp. 99-113, and pp. 137-172) there has appeared an enormous amount of programmed material in a variety of modes, and on a very wide range of topics.

33. From the beginning mathematics has been considered (by the promoters of programmed learning; not necessarily by mathematicians) as a suitable subject for programming, and programs on mathematical topics soon began to appear on the scene. I think that it is fair to say that mathematicians have largely been sceptical of the many optimistic claims which have been made for this new technique, and that May's two papers referred to above give a fair impression of the views of the mathematical fraternity. The reasons for these doubts are many: from the beginning the technique was oversold, both by educational enthusiasts and by commercial promoters; it was not at all obvious that work done on animals would have direct application to the learning of human beings; the style of production makes it very difficult for a mathematician to get a quick feeling for a program, as he can for a book, by skimming the content and sampling a few passages; and when he took the trouble – an extremely boring procedure – to examine carefully a programmed mathematics text by following it through in detail, he was often appalled by the mathematics he found. (This is not very surprising, since most of the early mathematics programs were written by people with no established standing as mathematicians or as mathematics teachers – people who would have been most unlikely to write a normal mathematics text). In any event, mathematicians have a healthy cynicism with regard to any notion that the learning of mathematics can be made quickly and dramatically easier. This long-standing attitude is typified by Euclid's oft-quoted reply to King Ptolemy when asked to simplify the learning of geometry, "there is no Royal road to geometry".

34. The doubts of mathematicians concerning the rather extravagant claims of some of the early promoters of programmed learning, resulted in their determination to try out the medium for themselves. As a result many very substantial projects have been carried through in the preparation and testing of mathematics programs. I shall mention a few of them.

35. In 1961, the School Mathematics Study Group inaugurated an experimental "Programmed Learning Project", designed to produce and test a variety of programs on a single clearly-defined course (beginning algebra) for which S.M.S.G. had already prepared and tested two different standard textbooks. This project has been fully reported. (See[17]; this report also contains a very useful *Manual for Programmers).*

The writing teams contained experienced mathematics teachers and university mathematicians, as well as psychologists who were experts in programming. Initially two programs were prepared, using the Skinner (constructed response) and Crowder (multiple choice) modes. It is interesting to note that a book of about 500 pages in standard textbook form, occupied over 1,700 pages when programmed in "constructed response" form, and over 2,300 pages in "multiple choice" form. A later "hybrid" form (using both of the above forms, together with some passages of standard text) occupied about 1,000 pages. This development of a hybrid form resulted from experiences during the project in the use of the older forms, especially the apparent need to overcome the tendency to boredom caused by the unrelieved use of a single form. Hybrid programming can be considered as an S.M.S.G. contribution to the art of programming. The results of the S.M.S.G. experiment are given in detail in the report cited. Roughly speaking, they suggest that (insofar as these things can be measured), a well prepared program can be a useful learning device, and that its effectiveness as compared with the use of a standard textbook is likely to be neither dramatically better nor dramatically worse, when used in a reasonably sensible way. Another conclusion was that the hybrid form was more effective than either of the "pure" forms, giving the students more confidence in the use of terminology as well as improved performance, and giving greater satisfaction to the program writers. Perhaps the most significant single conclusion to be drawn from the S.M.S.G. study is that programmed learning deserves, and needs, a great deal more investigation and experimentation, in order to determine the best ways of combining different modes (including possible new modes) in order to maintain the student's interest and challenge his ability, as well as to condition him to certain desired responses. There is, of course, much more of value in the report, which I strongly recommend to anyone seriously interested in the use of programmed instruction for the improvement of school mathematics.

36. Programmed learning has mushroomed enormously in its very short span of existence, and while it has not lived up to some of the claims of the early enthusiasts, there is little doubt that it has made (and will continue to make) a worthwhile contribution to our armoury of learning resources, and to our understanding of the learning process. A great deal of experimental work is now in progress, but much more will need to be done before the full potentialities of this medium are realised. But it is safe to conjecture that it will eventually establish itself as one of many useful resources, and not, as some of its early promoters saw it, as the solution to many of our problems of teacher supply and training. All the evidence to date suggests that programmed material will only be an aid to the mathematics teacher, and will not supplant him.

37. This is all that I wish to say in order to introduce this topic, which will, no doubt, be widely discussed in our working parties. But before going on, I should point out that programmed learning is strongly linked with computer-assisted learning, about which I shall have more to say later in this paper. I conclude by giving a short reading list of some of the many books and papers which are relevant to the discussion of programmed learning in mathematics. To this list should be added the references already given, and the very useful reference lists which they themselves contain.

Leedham, J., and Unwin, D. *Programmed Learning in the Schools.* London, Longmans, 1965.

Fincher, G.E., and Fillmer, H.T. "Programmed Instruction in Elementary Arithmetic", *Arithmetic Teacher,* January, 1965.

Smith, M.D. "Some Considerations in Teaching Mathematics by Programmed Instruction", *Mathematics Teacher,* May, 1962.

Kalin, R. "Some Guidelines for Selecting a Programmed Text in Mathematics", *Mathematics Teacher,* January, 1966.

Smith, L.W. "The Use and Abuse of Programmed Instruction", *Mathematics Teacher,* December, 1965.

Heimer, R.T. "Designs For Future Exploration in Programmed Instruction", *Mathematics Teacher,* February, 1966.

School Mathematics Study Group. *Programmed First Course in Algebra,* (Student's Text, Parts I & II, Student's Response Booklet; Teacher's Commentary). New Haven, Yale U.P., 1965.

Mathematical Association of America, Committee on Educational Media. *A Programmed Calculus.* New York, W.A. Benjamin, 1968.

Television and Films

38. It is convenient to consider these two media together. Anything that can be put on film can be televised (including colour film on colour television – at a price!); and any television program can be recorded on film and used through a normal film projector. However, there are a number of important differences in the use of these media. Films have the potential for far greater visual quality; but television has the advantage in flexibility of transmission, and in permitting widespread simultaneous use – sometimes by millions of students at the same time, as happened with the well-known "Continental Classroom" series in the United States.

39. It is likely that every country which has public television has used (or will use) this medium at some time or other for educational purposes, and that this will include the teaching of mathematics. A few years ago some educational administrators were enthusiastically hailing television as the future "solution" to the teacher supply problem, but this has not happened. As far as mathematics is concerned, there are severe limitations to the use of television. By far the greatest amount of communication in the mathematics classroom is by means of a mixture of visual and aural stimuli, with the visual predominating. Except in the very simplest situations it is necessary to have in view (or available for being viewed at a glance) more visual information than can be carried on a standard (25 inch or less) television screen. And the use of some of the very large screen television projection devices which are now available (at considerable cost) does not help, because of the nature of the television picture with its relatively small number of picture lines to the frame – usually in the neighbourhood of five hundred to seven hundred. When this image is enlarged up to theatre-screen size the line spacing is increased, and in order to smooth out these discontinuities the viewer must increase his distance from the screen. All that is gained is the possibility of having many people view the same screen at once, instead of having a large class grouped for viewing around a number of small screens.

40. It is well known that seemingly irrelevant psychological factors can be very important in the learning process. For example, many educators have suggested that the effective use of television for educational purposes is hampered by the fact that we are conditioned to its dominant use for entertainment. It would be interesting to see careful experimentation in the use of television for education, in societies that have not already been "contaminated" by its use for other purposes. This is not to suggest that the use of this medium for its present (largely entertainment and advertising) purposes might be curtailed, but if we are to try to understand the potentialities of television for education we might learn something from the study of the "extreme" situation in which it is not used for any other purpose.

41. Like many of you, we in Australia have experimented with the production and use of mathematics programs on television. It is possible by careful planning to adjust to the severe restriction on written visual material, but I do not know of any television teacher of mathematics who did not feel that this limitation hampered his effectiveness. The limitation becomes more serious as the level of instruction increases, and at the tertiary level (where one frequently covers many chalkboards with a single mathematical development, with frequent reference back to earlier parts of the development) the medium has very little value at all.

42. In considering the potentiality of television for mathematics instruction the limitation of writing space is the aspect which immediately strikes the mathematics teacher, and this is probably the single most important reason for the lack of enthusiasm which most mathematicians have for the medium. It is instructive to compare this with the enthusiasm of some of our scientist colleagues, and to remind ourselves of some of the reasons. Scientists like television for its ability to show what is happening in some inaccessible place; for its ability to magnify for the class some detail of an experiment, such as the behaviour of bacteria as viewed under a microscope; for its ability to provide a close-up picture in a potentially dangerous situation – as in chemistry, or in dealing with radioactive materials; for its ability to show what is happening in an environment which must be kept sterile – as in a surgical operation; and so on. But none of these special uses is relevant to mathematics: a mathematical derivation or formula is not dangerous; it does not need to be kept in a sterile environment; and it gains nothing in logical clarity by being magnified or viewed in close up. Thus not only does television "cramp the style" of a mathematics teacher, but there is no easy way in which the mathematics teacher – in contrast to the teacher of science – can significantly exploit many of the potentialities of the medium.

43. This is not to say that television has no place in the teaching of mathematics. Much elementary mathematics teaching is related to the real world. Demonstrations with concrete materials, relevant experiments, animation, and so on, can be recorded and used via television (just as they can via film) in order to supplement classroom experiences. When used for this purpose the television picture loss in quality, as compared with film, but it has the advantage of being able to show a situation in real time (i.e. while it is happening); and it has the advantage in the ease with which the number of simultaneous viewers can be multiplied indefinitely, using closed circuit or broadcast transmission.

44. A few years ago, in the State in which I live, a new mathematics program was being introduced in the secondary schools. With the technical assistance of the Australian Broadcasting Commission, our Mathematical Association produced a series of thirty half-hour television programs. Each of these was broadcast to schools on three separate occasions, so that the schools could choose their own viewing times. These programs were ostensibly directed at students. But it was the opinion of most of those who were associated with this project that its greatest value lay in the assistance which it gave to teachers, as indirect inservice training. A corresponding conclusion has been reached by others in similar situations.

45. Looking ahead, it seems safe to say that the usefulness of television for allowing large numbers of simultaneous users, for the exploitation of visual material other than mere writing or printing, and for teacher inservice training, together with the possibility of recording and re-using material, will result in a steadily increasing use of the medium as an aid to the learning of mathematics. It is to be hoped that this use will be both critical and selective, and that ways might be found to exploit the real potentialities of the medium – those things which can be better done by television than in any other way.

46. A useful overview of the use of television in mathematical education in the United States is given in a recent report[18] of a conference on "Television in Mathematics Education", sponsored by the National Center for School and College Television. After viewing much of the very considerable volume of recorded television material (for mathematics) which is available in the United States, it was the opinion of the viewing panel that negligible use had been made of the real potentialities of the medium and that, on present evidence, television should not be used as a basic resource for mathematics instruction, but only as a special purpose supplementary resource. But there is much more to this report, and I strongly recommend that you read it yourselves.

47. There is one further use of television that I wish to mention. This is as an aid in the pedagogic training of teachers. A couple of years ago, when at Stanford University, I saw relatively cheap videotape recording and playback equipment being used in connection with teacher training, in much the same way that tapes are used in a language laboratory. Videotape was used to record practice lessons, or parts of lessons. (The "students" were fellow trainees, but real classes could be used, with some modifications). These were then played back and discussed and criticised in a seminar situation. Playback could be immediate; it could be stopped at any point, repeated, and so on. This appealed to me as an immensely valuable tool for teacher training. Most of us are rather appalled at the quality of our own voice on a tape recorder. I am sure that we would be even more humbled by the pedagogic deficiencies which would be disclosed by videotape recordings of our class teaching.

48. With films, as with television, it is relatively easy to record a standard classroom situation, and this has been done extensively. Most of the offerings in the 1963 listing and review[19] of mathematics films, and in the 1967 listing[20] of broadcast television offerings in mathematics (both sponsored by the National Council of Teachers of Mathematics) are of this type. But this simple and direct approach makes no significant use of the real potential of the film medium, and a number of

projects have been undertaken in an effort to do better. The common experience of most of the groups which have attempted to come to grips with this problem is that it is enormously difficult and costly to produce a really satisfying product.

49. One of the most informative, and best documented[21] film projects is the so-called "Level I Project" of the Mathematical Association of America's Committee on Educational Media. The original objectives of this project were to prepare 50 to 60 forty-five minute films which would provide the major expository portion of an undergraduate college course in number systems, designed primarily for use in the pre-service training of elementary school teachers of mathematics. The films were to be accompanied by text materials, problem sets, sample examinations, programmed material, and so on, (a true multi-media approach), and to be supplemented by classroom discussions, problem sessions, and examinations. Every effort was to be made to "use the potential of the film medium for animation and special effects" and due attention was to be paid to "the importance of the individual teacher, whose personality and enthusiasm must shine out through the film in a way that it cannot in a text".

50. As the project developed, from the summer of 1964, the members began to appreciate the enormity of the task they had undertaken. The original scope of the project was significantly reduced, and it was only due to the dedicated efforts of a few individuals – such well known men as Carl Allendoerfer and Julius Hlavaty – that the reduced project was brought to completion. During the Conference I hope to show some of the films (which are available for purchase or for rental: see[22]) from this project.

51. The report of this project should be required reading for any group contemplating the production of mathematics films and film strips as part of a multimedia approach to the teaching of mathematics. While the diminished project was, in a sense, completed, the work done suggested many more lines of activity which could not be pursued because of limited resources. No attempt was made to determine the best mix of the media used (films, film strips, text, programmed material, live teacher) – indeed the report suggests that what is "best" is likely to vary from class to class, and even from student to student. As far as films are concerned, a major conclusion of the report is that the cost of making a polished production of a full series of long films (essentially as filmed lectures, supplemented with various props) would be very high, and that the money could be better spent in other ways. On the other hand, some value is seen in the use of shorter films for motivation, and as aids to the intuition; but the author of the report is careful to point out that this tentative conclusion has not been carefully and objectively evaluated. Animation is seen as highly suitable for most mathematics films; but films of "stand-up lectures" are seen as largely ineffective for the learning of mathematics, although they might be of great value for other – e.g. archival – reasons. (Consider the value of a film showing Gauss or Newton – or even, alas, Einstein – as a teacher).

52. The M.A.A. Project is a pioneer in its attempt to exploit the full potentialities of the film medium, as part of a multi-media approach. There is no doubt that much more will be heard about this type of approach; but it is already clear

that a great deal of work will have to be done before we are in a position to understand and exploit it fully.

Radio

53. A few months ago I was aware of only two recent projects concerning the use of radio as a resource for learning mathematics. The first (which is run by the University of New South Wales, using its campus radio station) has two components: one of these is a remedial course designed to bridge some gaps in the mathematical education of students who are about to enter the University; the other is an in-service course for teachers. The broadcasts are keyed to written materials (which are sent out in advance to enrolled students), and it is not likely that they have any value for those not in possession of the written materials. My colleagues in Sydney seem to be reasonably satisfied that these programs are useful for the special purposes for which they are planned: that is, to reach a large audience at very low cost; and to give them a somewhat more personal stimulus than they would obtain from using written materials alone. Except for the difference in cost, this two-media approach (radio and keyed text) would seem to have no advantages over television, and this is, no doubt, the reason why this sort of program has received very little attention.

54. A second example of a project which uses radio and supplementary written materials, is the fairly recent introduction, in Australia, of "Schools of the Air". In the state of Western Australia about 500,000 square miles (roughly half of the area of the state) is covered by broadcasts from four transmitting school centres. These broadcasts are designed to assist the correspondence school education of children who live in such isolation that there are not enough of them living in an area of many thousand square miles to justify the establishment of even a one-teacher primary school. In addition to the correspondence work (which has been operated by the State Education Department over a long period of time), "School of the Air" broadcasts (using 2-way radio transceivers) were begun in 1959, in conjunction with the use of the same radio equipment as part of the Royal Flying Doctor Service. These broadcasts, which run for 2½ hours on each school day, cover the whole of the primary school program, including mathematics. Their two-way character is an important part of the project, permitting a significant amount of direct "feed back" from student to teacher. As the number of students at any one point (usually a cattle station) is generally less than five, the per student capital cost of the equipment is relatively high. But the cost is partly justified by the parallel use of the equipment for medical services. The whole operation covers about 200 students spread over seven grades. It involves about 10 teachers, who also handle the correspondence work. Some of the other Australian states have similar projects.

55. Just recently I came across another report[23] of an experiment in the use of radio as an aid in the teaching of mathematics in Wisconsin, U.S.A. This experiment consisted of a fifteen-minute broadcast, not keyed to any particular written material (i.e. it was self-contained), and neither the topic nor the script was available in advance. The experimenters reached the conclusion (based on replies to a questionnaire) that this procedure could be a useful supplement to the regular in-class teaching program. It is difficult to see that radio, as such, played any significant

role in the experiment; possibly the same result could have been achieved by the use of a tape or phonograph recording.

Structured Aids, Analogy Devices and other Classroom Materials

56. The materials which come under this heading are, presumably, covered by the "etc." in my terms of reference. Their relevance is greatest at (but by no means confined to) the primary level, and for this reason I have not had any significant involvement with any of them, except, of course, those which we also use extensively at the tertiary level. (e.g. chalkboards, computers and other calculating devices, overhead projectors). I am sure that many of you will be much more familiar than I am with the ever-growing variety of structured aids now available. Cuisenaire rods, Stern materials, attribute blocks, Dienes' multibase arithmetic blocks, pegboards, number balances, mirror cards, discovery boards, geoboards, abaci, slide rules, desk calculators, and so on. Many of these devices have their origin in the imaginative ideas of gifted teachers, and in the discoveries and theories of child psychologists concerning the learning processes of children. In addition to the well known (and commercially available) devices, there are undoubtedly thousands of home made and readily available materials which are being exploited by imaginative teachers as aids in the learning of mathematics. The Nuffield Mathematics Project[12] places great emphasis on the use of commonly available resources in the school and in the community: a great many of the "mathematics tasks" designed by its teachers for the early (and for the not-so-early) school years, involve the handling of physical materials. Many of the leaders in mathematics education for the primary grades (e.g. Robert Davis[4]) believe that the Nuffield Project is carving out a path which will be increasingly followed in the future. This is not really surprising if we believe that the environment in which we live has present significance in relation to personal mathematical development, as well as its well known historical role in the development of mathematics.

57. It seems safe to predict that, along with an increasing use of the environment (possibly through integrated science and mathematics programs), we will see an increasing use of specific, carefully devised, analogy-devices, such as the Stern, Cuisenaire, and Dienes materials, and that the most effective teaching will make use of all of these and the many other teaching aids which are, or will become, available. It is most unlikely that the path to success will lie in the exclusive use of any single device.

58. I conclude this section with a short reading list of some of the better known books on the use of concrete materials:

Stern, C. *Children Discover Arithmetic.* London, Harrap, 1958.

Stern, C. *Experimenting with Numbers.* New York, Houghton Mifflin, 1954.

Cuisenaire, G., & Gattegno, C. *Numbers in Colour.* London, Heinemann, 1954.

Dienes, Z.P. *Building Up Mathematics.* London, Hutchinson, 1960.

Dienes, Z.P. *The Power of Mathematics.* London, Hutchinson, 1963.

Dienes, Z.P. *An Experimental Study of Mathematics Learning.* London, Hutchinson, 1963.

Dienes, Z.P. *Mathematics in the Primary School.* Melbourne, Macmillan, 1964.
Sealey, L.G.W. *The Creative Use of Mathematics in the Junior School.* Oxford, Blackwell, 1960.

Computer-Assisted Learning

59. No discussion of the resources for learning mathematics can ignore the exciting new developments of the last few years in the use of computers as learning aids. This is the second way in which computers are going to have a major impact on mathematics education. (The first, which will undoubtedly be discussed in other plenary sessions, is the impact of the computer on the curriculum itself).

60. Among the best known experimental projects which use digital computers as teaching aids are those of Dr. Patrick Suppes, at Stanford; and Project Plato, at the University of Illinois. (See Reference [4]). Roughly speaking, the computer is used as a programmed learning device. A large time-shared computer can be programmed to give a degree of flexibility which would require many thousands of pages of printed matter, and which would be forbiddingly difficult to follow. The computer can make virtually instantaneous shifts to different parts of the program, guided by the most recent answer which the student gives (or even by a large number of previous answers). Moreover the growing capacity of computers makes it possible to envisage (as in Project Plato) as many as 6,000 simultaneous users (located in schools, universities, or even in their own homes and offices) of the one program on the same computer. And while doing all this the computer can store complete records of the responses of every individual, and print out these (or any pre-determined analysis of them) on command.

61. Thus the computer holds out the prospect of providing a vastly greater individualisation of teaching, limited only by the capacity of the computer and by the imagination and skill of the teacher-programmers. Moreover it promises to give the teacher an almost unlimited capacity for keeping records in immediately accessible form on each individual student. While it is not likely that any foreseeable computer will achieve the flexibility of response of a superior teacher who has only one student, computer-assisted instruction holds out great promise for significant progress in the direction of tailoring the education of each individual student to suit his own capabilities and his personal requirements; and, properly programmed, it can even assist in determining these capacities and needs.

62. At the present time the work being done is almost entirely experimental, and the cost is high. But this is falling steadily with technological advances, and there is little doubt that computer-assisted learning will be increasingly used in our educational systems. At least, as Robert Davis has pointed out, this seems to be the view of several of the large American companies with computer interests, many of whom have recently bought, or merged with, book publishers and manufacturers of other educational materials.

Evaluation

63. Evaluation of the effectiveness of any educational program, curriculum or learning resource is enormously difficult, and yet enormously important. Probably

no one has been more aware of the need for some measures of effectiveness than Dr. E.G. Begle, Director of the School Mathematics Study Group. Yet in a recent issue (March, 1968) of *The Mathematics Teacher,* in reference (inter alia) to investigations by his own group of the effectiveness of a programmed algebra course, he writes "None of the above can be considered to be properly designed and executed experiments ... important variables were either ignored or left unmeasured ... control over treatments was not always feasible ... the purpose of the studies was to obtain rough estimates... ."

64. It is safe to say that the same comments could be made about virtually every effort which has been made to evaluate the effectiveness of new curricula, or the usefulness of the growing number of teaching aids and other learning resources. Some idea of the difficulty and the cost of a fairly well designed and well executed experimental program can be gauged from two recent major studies concerned with mathematics education. One of these is the twelve-country UNESCO-sponsored "International Study of Achievement in Mathematics", whose two volume report[24] is notable for what it discloses about the difficulties and shortcomings of such a venture, and for the modesty of its claims concerning what has been "proved". The other is the National Longitudinal Study of Mathematical Abilities, a project of the School Mathematics Study Group. This was designed (in 1962) as a five year study to assess the effectiveness of new mathematics curricula. The report of this massive project (involving well over 100,000 students, from some 1,300 schools, at a cost of millions of dollars) is awaited with great interest.

65. The lesson to be learnt from these major projects is *not* that evaluation is so difficult that we should not bother to attempt it. It is essential that those using new learning resources should adopt an experimental and critical attitude, and that they should attempt to form some idea of the effectiveness of the resource for them and for their own students; but at the same time they should be aware of the largely subjective character, and the limitations, of present methods of evaluation. In view of these limitations, it is particularly important to beware of those educational conservatives who refuse to participate in any kind of innovation until its effectiveness has been "proved". It is safe to say that absolute "proof" will never be possible, and yet innovations must be made, guided by the collective wisdom and experience of mathematicians, teachers and educational administrators.

Summing up

66. What does this all add up to? It seems to me that there will be a growing use of physical and other "real world" aids to instruction (both in the classroom and outside it) at the elementary level, and that this will be more closely associated than it has been with experiences in "science". This will not supplant the current and growing emphasis on "structure", but rather it will provide the concrete experiences from which to abstract, and it will provide that evidence of "usefulness" which most children seem to desire. At all levels this will be increasingly supported by a growing sophistication in the design of textbooks and programmed materials, and in the use of new and existing communications media of all types – teachers, books, overhead-projector transparencies, films and film strips, television, computers etc. I do not see any particular resource or medium as predominating, but rather that there will

be a wider and more carefully planned multi-resource and multi-media approach – the kind of approach that good teachers have always used with the limited resources and media available to them. Perhaps in some distant generation, when we might have reached agreement on the aims of mathematical education (individualised to the need, the ability and the personality of each student), and when we might have mastered the problems of measuring the effectiveness of different curricula, resources, and media in achieving these aims, we will be able to put our computers to work to devise and control the optimum educational mix for each student. But this possibility is so remote that we should not let it spoil our pleasant anticipation of many more Conferences such as this.

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REPORT OF WORKING GROUP B.3

Chairman: Professor R.S. Varma (India)

67. Resources for learning mathematics as discussed by Group B.3 are presented in the following order -

(a) Books
(b) Audio-Visual Aids – Films, Film Strips, T.V., Radio, etc.
(c) Programmed Learning
(d) Low Cost Teaching Aids and Demonstration Equipment
(e) Resources involved in Mathematical Education at different levels
(f) Some Additional Resources

Books

68. Some criticism was expressed as to the type and age of some of the textbooks in use in some developing countries. The general feeling was that within a region attempts should be made to use and if necessary to write textbooks written along modern lines appropriate to their needs, and it was pleasing to hear reports from some areas that this was already in hand, and was being successfully pursued. It was thought that each country should proceed as it thought fit, seeking help from the developed countries if necessary. There might well be a need for international co-operation, for example, in the dissemination of knowledge about available books and it was considered desirable to put on tour collections of new books which could be circulated from country to country and even within a country itself.

69. Members emphasized the need for varying categories of books:-

(i) Books meant for pupils. These could be either textbooks, or small topic books, or books of reference to which students could refer, as well as attractive reading books on mathematical topics.
(ii) There is need for books of reference for teachers both on the purely mathematical front and on the teaching front which would suggest such things as: (a) modern teaching – approaches, (b) suggestions for the making and using of apparatus.

The view was expressed that there was a place for books on modern methods of teaching mathematics as well as modern content.

70. The need for background books which would enrich the mathematical imagination of pupils and teachers was emphasized Books of this kind suitable for pupils below the standard of the ordinary level examination were now available.

Audio Visual Aids – Film, Film Strips, Television, Radio

71. The Oversea Visual Aids Centre (O.V.A.C.) Tavistock House, South Tavistock Square, London, W.C.1., was established some five years ago in order to disseminate

information about visual aids, their consideration and use. It publishes a bulletin, details of which can be had from the above address.

72. The use of slides, colour or otherwise, was discussed, but it was felt that in the context of mathematics teaching, a slide projector was of very limited use, and was in fact being superseded by the overhead projector. The advantages of the overhead projector were said to be:-

(a) That not only could straight pictures be put on, but a sequence of transparencies could be used to build up a composite picture as a lesson proceeds.

(b) The structure of such overhead projectors makes it possible for shadows of solid objects to be cast on the screen. In the teaching of motion geometry this would prove a very valuable visual aid.

73. The role of Television in mathematics teaching was discussed at some length. In this respect television is still evolving and this medium has not yet been fully exploited. That television certainly cannot replace a teacher but can help to compensate for the shortage of *qualified teachers* was emphasized.

Television can be used for the following purposes:

(a) To explore regions of mathematics which lend themselves *(or could lend themselves)* to visual interpretation, especially of a type that teachers cannot easily cope with for one reason or another. The means to this end are very varied but usually involve film animation, electronic wizardry, models and animated captions.

(b) To explore fresh areas of mathematics with which most pupils and many teachers might not be familiar. This often means re-interpreting rather difficult books and inviting university and college lecturers to the studio. Occasionally it involves extensive actuality filming (e.g. of computers in operation).

(c) Linked with (b) is the task to explore the mathematics all around by drawing on examples from everyday life and by showing applications in science, technology, sociology, etc. Apart from the techniques already mentioned, photographs are often of value here.

(d) To use the medium to "pipe" all kinds of mathematics to schools which are short of qualified staff. This can allow more effective use of the time of qualified teachers either by themselves or in some form of team-teaching.

(e) To encourage and advance the work of slow learning pupils. Programmes produced by the B.B.C. for slow readers using the full gamut of visual techniques have been particularly successful and it was thought that this might provide a parallel for the learning of mathematics by the slow learning pupils. One of the most surprisingly successful devices used for encouraging children's work at all levels of ability has been that of showing films of other children working.

(f) To inform teachers of developments in the subject matter and teaching of the subject using all the techniques mentioned above.

(g) To act as a source of secondary material in the form of books, films, etc.

74. Possible uses of video tape to record and supplement transmitted programmes were also considered.

75. In some areas the programmes were supplemented by correspondence courses based on the programme material.

76. This Committee could find very little experience of radio being used as an agency for mathematics teaching, but it was posed as a possibility that it might well be used in the context of the in-service training of teachers for critical discussions of television or other programmes.

77. It was felt that one of the advantages of using television and radio, was that the wider audience of parents and other interested parties could be reached as well as the pupils and teachers to whom the programme was really directed. This was considered a great advantage when considering any form of educational change whether of mathematical content or of approach.

78. Some interesting experiments were reported by way of two-way telephonic links sometimes supported by ingenious electronic visual devices, but it was felt that these had not yet realised their full potential and were very experimental in nature.

Programmed Learning

79. It was pointed out that the ideas underlying programmed learning had been a valuable teaching technique for many years. What, however, was new, was an influx of hardware and/or books which were specifically tightly programmed. The view was expressed that most of these approaches were often deadly dull in their impact on pupils, but nevertheless they may find a positive use particularly in remedial situations, e.g. to remedy a gap in learning due to absence or some such other cause. The point was made that no machine is better than the programme which is fed into it and it was felt that many of the commercial programmes did not measure up to our requirements. It was felt that at best such machines would facilitate the learning and practice of skills rather than the understanding of concepts.

Low Cost Teaching Aids and Demonstration Equipment

80. With regard to the production of low cost teaching aids, the first matter considered was what should be produced, and it was pointed out that mathematical teaching journals in the developed countries often contained articles and references about making and using such aids and demonstration equipment; for example, Mathematics Teaching No.18, Journal of the Association of Teachers of Mathematics (U.K.) is devoted to this. A further source would be the commercial catalogues from school equipment firms. It was felt that Ministries concerned in the various countries should encourage the local production of such teaching aids and equipment by whatever local means are found to be appropriate in their circumstances.

81. Some suggestions (minimal) with regards to equipment were made:-

(1) *Squared Paper:* Various problems would demand squared paper of different unit squares and, therefore, a *variety* of squared papers should be available.

A blackboard or other board ruled in squares to be used for demonstration purposes should also be available.

(2) *Physical Apparatus:* It was felt that some form of co-ordinates board was a very useful feature. Nails at the corners of congruent squares covering a board form a pin board (peg board, geo board, lattice board) which can be used throughout all school grades.

(3) *Squares and Cubes:* A large collection of squares and of cubes for use by pupils was essential. Some of these should be unit squares, e.g. square inch; and unit cubes, e.g. cubic inch.

(4) *Fraction-boards:* Various forms of fraction-boards for developing the concept of fractional parts.

This list must be regarded as an absolute minimum.

82. The items are capable of being made locally from existing materials and are, therefore, low in cost.

Resources Involved in Mathematical Education at Different Levels

83. It was pointed out that in the United Kingdom a project was in hand to study ways and means of organising work in schools so as to make the best use of teachers' skills and new developments in method and equipment. The project was established in 1966 and is financed by Nuffield Foundation in co-operation with the Schools Council. Further information can be obtained from Mr. T. McMullen, Tavistock House, South Tavistock Square, London, W.C.1.

The group felt the necessity to state priorities in matters of resources:

Primary School: the priorities were thought to be:-

(1) Qualified Teachers.

(2) Materials arising from environment. This variously could be sea shells, mangoes, etc.

(3) Books (text, reading, reference).

Secondary School: the priorities were ordered rather differently as:

(1) Qualified Teachers.

(2) Books (text, reading, reference).

(3) Physical Equipment.

Some Additional Resources

84. Buildings, desk calculators and computer assisted learning were mentioned, but could not be discussed for want of time.

85. The group was unanimous that the most important resource for teaching was the teacher.

PLENARY DISCUSSION OF REPORT OF WORKING GROUP B.3

Comments and Survey

86. As was pointed out by Professor Varma when presenting the report to the plenary session, so vast a topic resulted in the discussion covering only part of the field.

Resources could be seen to serve Primary Education, Secondary Education and Teacher Training. References to specific resources were contained in the reports of groups A.1, A.2 and B.2. This to some extent helped to narrow the discussions undertaken.

87. Attention was drawn to Modern Curriculum Developments in Britain published by CREDO in 1968. This was regarded as a useful source book for those seeking details of British educational projects, mathematical and otherwise.

88. Educational technology is advancing at a pace comparable to other technological developments. It was in the spirit of "What is new and strange today becomes an everyday affair very quickly", that delegates heard with interest of the "hardware", being introduced in some countries. For various reasons, including the economy of the country, the lack of electric power and the depredations of some tropical insects, many delegates felt that any immediate application of much of the "hardware" was not for them. It was, however, noted that in some developing countries, television had been introduced as an educational medium, and that several countries had instituted centres for audio-visual aids. It was not so much in regard to the media themselves but in the use made of them that fears were expressed. These fears could be placed in two classes:-

(i) That available material, either good or bad, suitable or unsuitable, would be uncritically accepted in the schools. Such uncritical acceptance would not be in the best interests either of the pupils or the teachers. The same can of course be said for the manner in which some textbooks are used or misused.

(ii) That the radio or television voice would be regarded as "the voice of authority", and some teachers would tend to sit back and "let the machine do the work". The teacher being replaced by the machine was an often expressed fear; but as one delegate remarked "The teacher who can be replaced by a machine deserves to be replaced by a machine".

The remedies for these doubts would appear to be:-

(i) That teachers themselves control the content of the programmes. This will later be discussed at greater length when considering television.

(ii) That by initial and in-service training, teachers can be helped to use modern devices wisely as productive tools of their trade rather than as ends in themselves.

It was thought that in mathematics teaching a multi-media approach was the desirable one; it was for each country to find ways of making optimum use of all the resources available to it.

A detailed historical account of developments in the U.S.A. will be found in *A History of Instructional Technology,* Paul Seitler, McGraw Hill (1968).

Note was made of the existence in the United Kingdom of the *National Council of Educational Technology* operating from 160 Great Portland Street, London, W.1. One of its functions is to bridge whatever gap there may be between the users and the makers of such teaching aids.

89. *Desk Calculators* were regarded as desirable but expensive. As calculating devices, many felt that it would be a very long time before they would supersede the slide-rule or the logarithm table. More hope was held out for them as teaching aids in the primary and lower secondary school. It was claimed that as well as motivating learning, the pupils were helped in grasping the concept of place value; that the relationships between addition and subtraction, addition and multiplication, subtraction and division were clarified, in that they were operative procedures on the machine.

90. *Computers and Computer-assisted Learning.* Where computer studies had been introduced the interest of the students was phenomenal. Computer science experiments were reported from Ontario, Wales and England. In the case of the Welsh experiment, a telex link had been made between the school and a university computer. The aim of the operation was twofold:-

(i) Using the computer as a computing tool at the VI form (6th and 7th year secondary) level.

(ii) Helping pupils of all ability ranges to understand the applications, implications and limitations of computers rather than to lead pupils toward computer programming.

It was thought that work on computer assisted learning was not yet sufficiently developed for its worth to be properly evaluated. Some fears were expressed that we might revert to the sort of teaching and learning situation from which we are now struggling to escape. Care would be needed lest a straitjacket of one pattern be replaced by a straitjacket of another pattern. Even so, it was felt that the method might have considerable potential for certain aspects of mathematics.

91. *Television* for schools has two separate connotations:-

(a) Programmes broadcast on open national networks.

(b) Closed circuit television within a college or school, a group of schools, or perhaps throughout the schools of a particular area. A number of teacher training colleges in Britain are equipped with closed circuit television, as are schools in Glasgow, Leicester, London and Plymouth, to name a few authorities using this medium.

In the second case the responsibility for content, programme making, production and transmission are entirely in the hands of the teachers and educational advisers concerned. Such a set up should allay the fears of those who are critical of programmes being imposed from outside the schools.

On the national networks in the United Kingdom, the general policy for school programmes and the scope and purpose of each series of programmes are laid down by councils on which sit representatives of the professional associations of teachers, local education authorities, the Department of Education and Science and other educational organisations. Production of programmes and transmission are matters for the broadcasting authorities. Practising teachers often present the programmes. The teacher links here are again strong.

The conference background paper *The Role of Television in Mathematical Education* gave an exposition in some detail of the current situation in Britain. It also discussed some detail of the *Maths Today* series of the B.B.C. The 20 minute programmes are aimed at first and second year secondary pupils and each programme is transmitted at four different times in a fortnight; side by side with these programmes for the pupils are fortnightly supporting programmes for teachers in the *Teaching Maths Today* series. This work is further supported by the written word in the form of guides for the teachers and work/study sheets for the pupils. All this can be augmented by 8 mm. film loops in cassettes which are on sale. Evaluation of such a multi-media approach has yet to be made.

It would be difficult either for teacher or pupil to be a non-participant in such a widely based operation. Participation was perhaps even easier in a Scottish experiment, reported by a delegate where the television programmes consisted of direct teaching to pupils. The programmes were interrupted at appropriate points – at a question perhaps, or a point for brief discussion with the teacher in the room. During these short breaks the teacher took over, and then the programme proceeded to the next phase. The delegate continued "What these programmes have done to the teachers has been remarkable – it is perhaps the best training device I have ever seen. What it has done for the children I shall know next year".

92. *Film loops* were thought to have considerable potential in that there was flexibility in the selection to be made and they were less expensive than long films. Furthermore, individual work assignments at varying levels could be based on a suitable film loop. Film loops were thought more useful than a normal film, which was considered to be better from the background or enrichment point of view than from the direct learning viewpoint.

93. It was natural that the newer and perhaps therefore more exciting forms of educational tool should, in such free discussion, take precedence over the more traditional, humbler, less expensive tools such as books, blackboards, environmental and other physical materials. In the realistic spirit of the conference, it was felt that for many countries, ways and means of making optimum use of the resources currently available to them should continue to exercise all their powers. The lead paper and report under discussion rightly emphasized these humbler features. but perhaps equally rightly the discussion complemented this by dealing with the more novel resources.

Since learning is an individual affair, any artefact which will stimulate and ensure sound individual learning is worthy of serious consideration by all educators.